

applying the hash function on the message to produce the series of  $k$  values  $b_1, \dots, b_k$ ; and

verifying that the equations  $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$  are satisfied.

Kindly amend claim 4 as follows:

4. (Amended) A method according to claim 1 and wherein the method comprises an HFEV scheme and the set S2 comprises [the] a set  $f(a)$  of  $k$  polynomial functions of the HFEV scheme.

Kindly amend claim 5 as follows:

5. (Amended) A method according to claim 1 and wherein the method comprises a UOV scheme and the set S2 comprises [the] a set  $S$  of  $k$  polynomial functions of the UOV scheme.

Kindly amend claim 6 as follows:

6. (Amended) A method according to claim 1 and wherein said supplying [step] comprises [the step of] selecting the number  $v$  of "vinegar" variables to be greater than the number  $n$  of "oil" variables.

Kindly amend claim 7 as follows:

7. (Amended) A method according to claim 1 and wherein  $v$  is selected such that  $q^v$  is greater than  $2^{32}$ , where  $q$  is the number of elements of a finite field  $K$  over which the sets S1, S2 and S3 are provided.

Kindly amend claim 8 as follows:

8. (Amended) A method according to claim 1 and wherein said supplying [step] comprises [the step of] obtaining the set S1 from a subset S2' of  $k$  polynomial functions of the set S2, the subset S2' being characterized [by] in that all coefficients of components involving any of the  $y_1, \dots, y_k$  variables in the  $k$  polynomial functions  $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$  are zero, and the number  $v$  of "vinegar" variables is greater than the number  $n$  of "oil" variables.

Kindly amend claim 9 as follows:

9. (Amended) A method according to claim 8 and wherein the set S2 comprises [the] a set S of k polynomial functions of [the] a UOV scheme, and the number v of "vinegar" variables is selected [so as] to satisfy one of the following conditions:
- (a) for each characteristic p other than 2 of a field K in an "Oil and Vinegar" scheme of degree 2, v satisfies the inequality  $q^{(v-n)-1} * n^4 > 2^{40}$ , where K is a finite field over which the sets S1, S2 and S3 are provided.
  - (b) for p = 2 in an "Oil and Vinegar" scheme of degree 3, v is greater than  $n*(1 + \sqrt{3})$  and [lower] less than or equal to  $n^3/6$ , and
  - (c) for each p other than 2 in an "Oil and Vinegar" scheme of degree 3, v is greater than n and [lower] less than or equal to  $n^4$ .

Kindly amend claim 10 as follows:

10. (Amended) A method according to claim 8 and wherein the set S2 comprises [the] a set S of k polynomial functions of [the] a UOV scheme, and the number v of "vinegar" variables is selected [so as] to satisfy the inequalities  $v < n^2$  and  $q^{(v-n)-1} * n^4 > 2^{40}$  for a characteristic p=2 of a field K in an "Oil and Vinegar" scheme of degree 2, where K is a finite field over which the sets S1, S2 and S3 are provided and q is the number of elements of K.

Kindly amend claim 15 as follows:

15. (Amended) In an "Oil and Vinegar" signature method, an improvement comprising [the step of] using more "vinegar" variables than "oil" variables.

Kindly amend claim 16 as follows:

16. (Amended) A method according to claim 15 and wherein a [the] number v of "vinegar" variables is selected [so as] to satisfy one of the following conditions:
- (a) for each characteristic p other than 2 of a field K and for a degree 2 of the "Oil and Vinegar" signature method, v satisfies the inequality  $q^{(v-n)-1} * n^4 > 2^{40}$ , where n is a number of "oil" variables, K is a finite field from which the n

"oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K.

(b) for  $p = 2$  and for a degree 3 of the "Oil and Vinegar" signature method,  $v$  is greater than  $n \cdot (1 + \sqrt{3})$  and [lower] less than or equal to  $n^3/6$ , and

(c) for each  $p$  other than 2 and for a degree 3 of the "Oil and Vinegar" signature method,  $v$  is greater than  $n$  and [lower] less than or equal to  $n^4$ .

Kindly amend claim 17 as follows:

17. (Amended) A method according to claim 15 and wherein [the set S2 comprises the set S of k polynomial functions of the UOV scheme, and the] a number v of "vinegar" variables is selected [so as] to satisfy the inequalities  $v < n^2$  and  $q^{(v-n)-1} * n^4 > 2^{40}$  for a characteristic  $p=2$  of a field K in an "Oil and Vinegar" scheme of degree 2, where n is a number of "oil" variables, K is a finite field from which the n "oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K.

Kindly add the following new claims:

--18. A signature generator comprising:

a signature input receiver operative to receive a set S1 of k polynomial functions as a public-key and a message to be signed, the set S1 including the functions  $P_1(x_1, \dots, x_{n+v}, y_1, \dots, y_k), \dots, P_k(x_1, \dots, x_{n+v}, y_1, \dots, y_k)$ , where  $k, v$ , and  $n$  are integers,  $x_1, \dots, x_{n+v}$  are  $n+v$  variables of a first type,  $y_1, \dots, y_k$  are  $k$  variables of a second type, and the set S1 is obtained by applying a secret key operation on a set S2 of k polynomial functions  $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$ , where  $a_1, \dots, a_{n+v}$  are  $n+v$  variables which include a set of  $n$  "oil" variables  $a_1, \dots, a_n$ , and a set of  $v$  "vinegar" variables  $a_{n+1}, \dots, a_{n+v}$ ; and

a signature processor operatively associated with the signature input receiver and operative to perform the following operations:

to apply a hash function on the message to produce a series of  $k$  values

$b_1, \dots, b_k$ ,

to substitute the series of  $k$  values  $b_1, \dots, b_k$  for the variables  $y_1, \dots, y_k$  of the set  $S_2$  respectively to produce a set  $S_3$  of  $k$  polynomial functions  $P''_1(a_1, \dots, a_{n+v}), \dots, P''_k(a_1, \dots, a_{n+v})$ ,

to select  $v$  values  $a'_{n+1}, \dots, a'_{n+v}$  for the  $v$  "vinegar" variables  $a_{n+1}, \dots, a_{n+v}$ ;

to solve a set of equations  $P''_1(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v}) = 0, \dots,$

$P''_k(a_1, \dots, a_n, a'_{n+1}, \dots, a'_{n+v}) = 0$  to obtain a solution for  $a'_1, \dots, a'_n$ ; and

to apply the secret key operation to transform  $a'_1, \dots, a'_{n+v}$  into a digital signature  $e_1, \dots, e_{n+v}$ .

19. Apparatus according to claim 18 and also comprising a signature verifier operatively associated with the signature processor and operative to verify the digital signature.

20. Apparatus according to claim 19 and wherein said signature verifier is operative to verify the digital signature by performing the following operations:

obtaining the signature  $e_1, \dots, e_{n+v}$ , the message, the hash function and the public key;

applying the hash function on the message to produce the series of  $k$  values  $b_1, \dots, b_k$ ; and

verifying that the equations  $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$  are satisfied.

21. Apparatus according to claim 18 and wherein the signature processor is operative to perform an HFEV scheme, and the set  $S_2$  comprises a set  $f(a)$  of  $k$  polynomial functions of the HFEV scheme.

22. Apparatus according to claim 18 and wherein the signature processor is operative to perform a UOV scheme, and the set  $S_2$  comprises a set  $S$  of  $k$  polynomial functions of the UOV scheme.

23. Apparatus according to claim 18 and wherein the number  $v$  of "vinegar" variables is greater than the number  $n$  of "oil" variables.

24. Apparatus according to claim 18 and wherein  $v$  is selected such that  $q^v$  is greater than  $2^{32}$ , where  $q$  is the number of elements of a finite field  $K$  over which the sets  $S1$ ,  $S2$  and  $S3$  are provided.

25. Apparatus according to claim 18 and wherein the set  $S1$  is obtained from a subset  $S2'$  of  $k$  polynomial functions of the set  $S2$ , the subset  $S2'$  being characterized in that all coefficients of components involving any of the  $y_1, \dots, y_k$  variables in the  $k$  polynomial functions  $P'_1(a_1, \dots, a_{n+v}, y_1, \dots, y_k), \dots, P'_k(a_1, \dots, a_{n+v}, y_1, \dots, y_k)$  are zero, and the number  $v$  of "vinegar" variables is greater than the number  $n$  of "oil" variables.

26. Apparatus according to claim 25 and wherein the set  $S2$  comprises a set  $S$  of  $k$  polynomial functions of a UOV scheme, and the number  $v$  of "vinegar" variables is selected to satisfy one of the following conditions:

(a) for each characteristic  $p$  other than 2 of a field  $K$  in an "Oil and Vinegar" scheme of degree 2,  $v$  satisfies the inequality  $q^{(v-n)-1} * n^4 > 2^{40}$ , where  $K$  is a finite field over which the sets  $S1$ ,  $S2$  and  $S3$  are provided,

(b) for  $p = 2$  in an "Oil and Vinegar" scheme of degree 3,  $v$  is greater than  $n*(1 + \sqrt{3})$  and less than or equal to  $n^3/6$ , and

(c) for each  $p$  other than 2 in an "Oil and Vinegar" scheme of degree 3,  $v$  is greater than  $n$  and less than or equal to  $n^4$ .

27. Apparatus according to claim 25 and wherein the set  $S2$  comprises a set  $S$  of  $k$  polynomial functions of a UOV scheme, and the number  $v$  of "vinegar" variables is selected to satisfy the inequalities  $v < n^2$  and  $q^{(v-n)-1} * n^4 > 2^{40}$  for a characteristic  $p=2$  of a field  $K$  in an "Oil and Vinegar" scheme of degree 2, where  $K$  is a finite field over which the sets  $S1$ ,  $S2$  and  $S3$  are provided and  $q$  is the number of elements of  $K$ .

28. Apparatus according to claim 18 and wherein said secret key operation comprises a secret affine transformation  $s$  on the  $n+v$  variables  $a_1, \dots, a_{n+v}$ .

29. Apparatus according to claim 21 and wherein said set S2 comprises an expression including k functions that are derived from a univariate polynomial.
30. Apparatus according to claim 29 and wherein said univariate polynomial includes a univariate polynomial of degree less than or equal to 100,000.
31. A signature verifier for verifying the digital signature generated by the signature generator of claim 18, the signature verifier comprising a verifier processor operative to perform the following operations:
- to obtain the signature  $e_1, \dots, e_{n+v}$ , the message, the hash function and the public key via the signature input receiver;
  - to apply the hash function on the message to produce the series of k values  $b_1, \dots, b_k$ ; and
  - to verify that the equations  $P_1(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0, \dots, P_k(e_1, \dots, e_{n+v}, b_1, \dots, b_k) = 0$  are satisfied.
32. In an "Oil and Vinegar" signature generating apparatus an improvement characterized in that the "Oil and Vinegar" signature generating apparatus is operative to use more "vinegar" variables than "oil" variables.
33. An "Oil and Vinegar" signature generating apparatus according to claim 32 and wherein a number v of "vinegar" variables is selected to satisfy one of the following conditions:
- (a) for each characteristic p other than 2 of a field K and for a degree 2 of an "Oil and Vinegar" signature method, v satisfies the  $q^{(v-n)-1} * n^4 > 2^{40}$ , where n is a number of "oil" variables, K is a finite field from which the n "oil" variables and the v "vinegar" variables are selected, and q is the number of elements of K,
  - (b) for p = 2 and for a degree 3 of the "Oil and Vinegar" signature method, v is greater than  $n * (1 + \sqrt{3})$  and less than or equal to  $n^3/6$ , and
  - (c) for each p other than 2 and for a degree 3 of the "Oil and Vinegar" signature method, v is greater than n and less than or equal to  $n^4$ .